

DECLARATION:

My residence, post office address and citizenship are as stated below next to my name.

[_] was filed on ____ as Application Serial Number ____ and was amended on ____ (if applicable).

I acknowledge the duty to disclose information which is material to patentability in accordance with Title 37, Code of Federal Regulations, §1.56.

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PRIOR FOREIGN APPLICATIONS

Number	Country	Date Filed	Priority Claimed (Yes/No)
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I hereby claim the benefit under Title 35, United States Code §119(e) of any United States Provisional application(s) listed below.

APPLICATION NUMBER	FILING DATE
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I hereby claim the benefit under Title 35, United States Code, §120 of any United States application(s) listed below and, insofar as the subject matter of each of the claims of this application is not disclosed in the prior United States application in the manner provided by the first paragraph of Title 35, United States Code, §112, I acknowledge the duty to disclose material information as defined in Title 37, Code of Federal Regulations, §1.56 which became available between the filing date of the prior application and the national or PCT international filing date of this application:

PRIOR UNITED STATES APPLICATIONS

Application Serial Number	Filing Date	Status
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I hereby declare that all statements made of my own knowledge are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code and that such willful false statements may jeopardize the validity of the application or any patent issued thereon.


POWER OF ATTORNEY:

On behalf of Mitsubishi Electric Research Laboratories, Inc., Assignee of my entire right, title and interest, I hereby appoint the following attorney with full power of substitution to act exclusively for Mitsubishi Electric to prosecute this application and transact all business in the Patent and Trademark Office connected therewith: Dirk Brinkman, Reg. No. 35,460.

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Appendix A

Nonrigid affine correction

One way to estimate a correction matrix $\mathbf{J} \doteq \mathbf{M} \backslash \tilde{\mathbf{M}}$ generalizes the solution for the rigid affine correction given above. The strategy is to break \mathbf{M} into column-triples. Each column-triple is a stack of rotation matrices scaled by morph weights. Let $\mathbf{m}_{f_k,x}^\top, \mathbf{m}_{f_k,y}^\top \in \mathbf{M}$ be the x and y projections in frame f as given by column-triple k . As in the rigid affine correction, in a properly structured motion matrix \mathbf{M} these vectors should have equal norm and be orthogonal:

$$\forall_{f,k} \left[\|\mathbf{m}_{f_k,x}\| = \|\mathbf{m}_{f_k,y}\| \right] \wedge \left[\mathbf{m}_{f_k,x}^\top \mathbf{m}_{f_k,y} = 0 \right]. \quad (1)$$

Moreover, their projections onto vectors from other column triples should also have equal norm (because all column-triples have the same rotations):

$$\forall_{f,k,j} \left[\mathbf{m}_{f_k,x} \mathbf{m}_{f_j,x}^\top = \mathbf{m}_{f_k,y} \mathbf{m}_{f_j,y}^\top \right] \wedge \left[\mathbf{m}_{f_k,x}^\top \mathbf{m}_{f_j,y} = 0 \right]. \quad (2)$$

This yields a system of equations

$$\forall_{f,k,j} \left(\text{vec}(\mathbf{m}_{f_k,x} \mathbf{m}_{f_j,x}^\top - \mathbf{m}_{f_k,y} \mathbf{m}_{f_j,y}^\top) \right)^\top \text{vec} \mathbf{H}_{k,j} = 0, \quad (3)$$

$$\forall_{f,k,j} \left(\text{vec}(\mathbf{m}_{f_k,x} \mathbf{m}_{f_j,y}^\top) \right)^\top \text{vec} \mathbf{H}_{k,j} = 0. \quad (4)$$

Now recall that each $\mathbf{H}_{k,j}$ is the outer product of two column-triples in (\mathbf{J}^{-1}) , e.g.,

$$\mathbf{H}_{k,j} = (\mathbf{J}^{-1})_{\text{cols}(3k-2, 3k-1, 3k)} (\mathbf{J}^{-1})_{\text{cols}(3j-2, 3j-1, 3j)}^\top. \quad (5)$$

Consequently, the matrix

$$\mathbf{H} \doteq \begin{bmatrix} \mathbf{H}_{1,1} & \cdots & \mathbf{H}_{1,K} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{K,1} & \cdots & \mathbf{H}_{K,K} \end{bmatrix} = (\mathbf{J}^{-1})^{(3K,3)} (\mathbf{J}^{-1})^{(3K,3)\top} \quad (6)$$

should be symmetric with rank 3. Let $\mathbf{V} \mathbf{\Lambda} \mathbf{V}^\top \xleftarrow{\text{EIG}_3} \mathbf{H}$ be a truncated decomposition of \mathbf{H} using its three largest eigenvalues and their associated eigenvectors. Then the desired correction is $(\mathbf{J}^{-1}) = (\mathbf{V} \sqrt{\mathbf{\Lambda}})^{(3K,3)}$.

Although formally “correct,” this procedure is of limited use because in order to express eqns. (3–4) in terms of \mathbf{J}^{-1} we must make the substitution $\mathbf{m}_{f_k,x}^\top \rightarrow \tilde{\mathbf{m}}_{f_k,x}^\top (\mathbf{J}^{-1})_{\text{cols}(3k-2, 3k-1, 3k)}$, which makes the constraints on all $\mathbf{H}_{k,j}$ nearly identical. Consequently the linear system is rank-deficient, because the number of unknowns in \mathbf{H} grows as $O(K^4)$ (or $O(K^3)$ if one only considers $j = \{k, k+1\}$) while the number of true unknowns in \mathbf{J}^{-1} grows as $O(K^2)$. In practice, there are enough constraints to support a usable estimate of the first three columns of \mathbf{J}^{-1} . We can therefore calculate the first column-triple of $\tilde{\mathbf{M}}$, project $\tilde{\mathbf{M}}$ into the $3K - 3$ dimensional space orthogonal to this, and repeat the procedure to get the next column triple of $\tilde{\mathbf{M}}$. A generalized SVD solution for factoring \mathbf{H} without explicitly computing its elements (thereby avoiding the rank-deficient division) requires some extra pages to explain and therefore will be published separately.